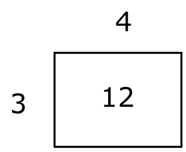
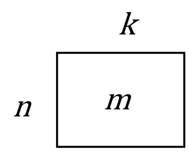
Asking for Arrays: $5 \div \frac{2}{3}$

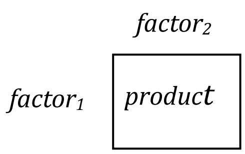
Every division relationship is also a multiplication relationship: $\frac{m}{n} = k \iff m = nk$

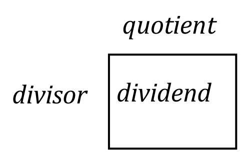
To say that some number *m* divided by some other (non-zero) number *n* equals some number *k* means that *m* is equal to *n* copies of *k*. Let's look at it concretely: to say that 12 ÷ 3 = 4 means that 3 x 4 = 12. We can picture this multiplication relationship as a rectangle, with the factors being the lengths of the sides and the product being the area, like this:





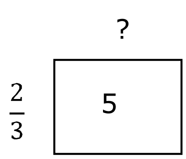
This area model works from a division point of view, too: we have the lengths of the sides being the quotient and the divisor and the number inside (the area) being the dividend:





This should remind us of the standard way of laying out a division problem: $\begin{array}{r} 4 \\ 3\overline{)\,12} \end{array}$

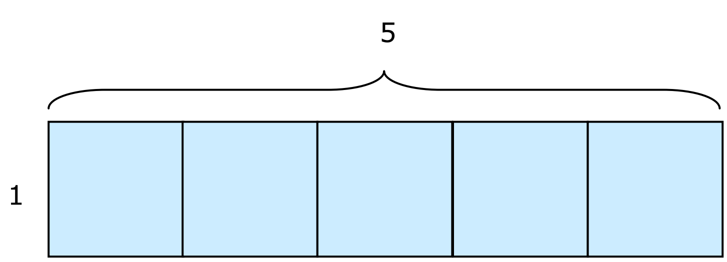
and since we can express any product of two factors as a rectangular array with the factors as its dimensions, then we ought to be able to diagram $5 \div \frac{2}{3}$ like this:



Building our rectangle:

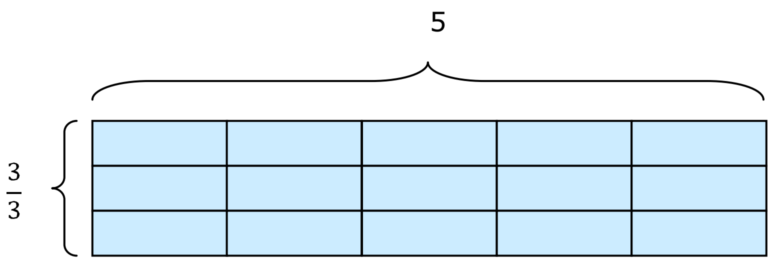
We know that $\frac{2}{3}$ times some number will give us 5, but we don't know what that number is yet.

We know that 5 x 1 gives us 5, which we can easily build:

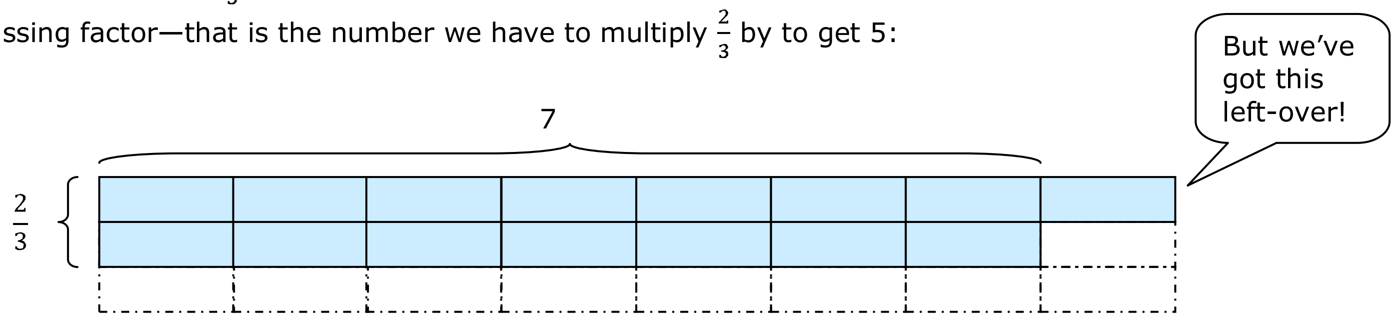


Transforming our 1 x 5 rectangle:

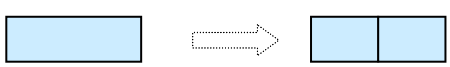
We've got our total area = 5 square units. But our height is too tall: we need $\frac{2}{3}$ as one of the factors, so we divide the height of our rectangle into thirds:



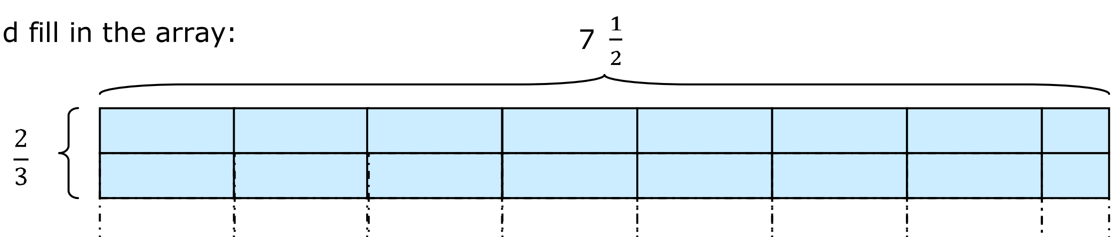
Then we re-arrange the pieces so the dimension on one side = $\frac{2}{3}$. As long as we don't take anything away, the area of the rectangle will be the same as our original 1 x 5 rectangle. So if we keep the height of the rectangle at $\frac{2}{3}$, whatever the other side turns out to be when we make a rectangle will be the missing factor—that is the number we have to multiply $\frac{2}{3}$ by to get 5:



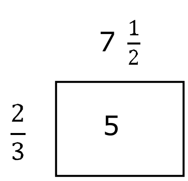
That $\frac{1}{3}$ x 1 left-over piece keeps us from having a real rectangle, so we divide it in half



and fill in the array:

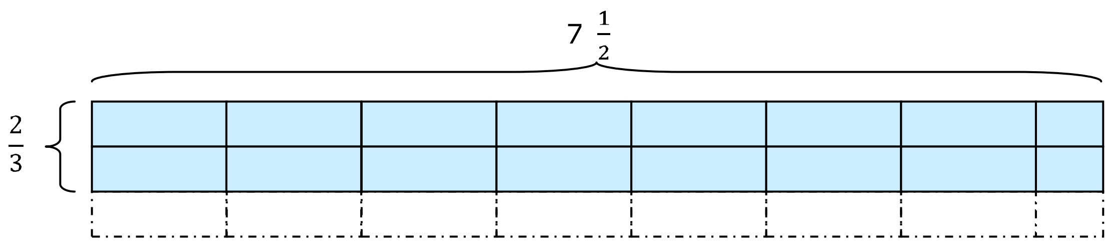


We haven't changed the total area—it's still 5 square units. The height is still $\frac{2}{3}$.
So the width (which is $7\frac{1}{2}$) must be the missing number in our relationship.
Now we can fill in our diagram:



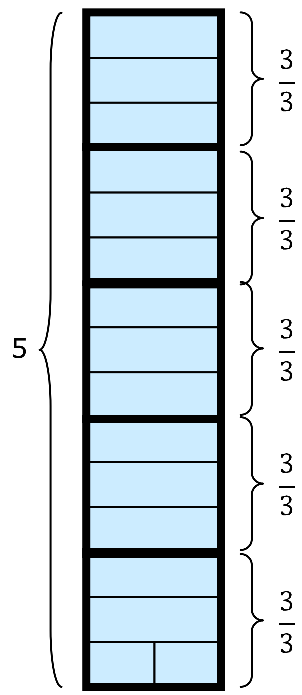
Checking our work:

$\frac{2}{3}$ x $7\frac{1}{2}$ could be diagrammed like this:



When we re-arrange them and stack all the $\frac{1}{3}$ x 1 rectangles, we have 14 little $\frac{1}{3}$ x 1 rectangles and, at the bottom, 2 even littler $\frac{1}{3}$ x $\frac{1}{2}$ rectangles, making a total of 15 little $\frac{1}{3}$ x 1 rectangles.

We can write that as $\frac{15}{3}$ x 1, which covers the same area as 5 square 1 x 1 units.



Working symbolically, it amounts to the same thing:

$$
\begin{aligned}
7\tfrac{1}{2} \bullet \tfrac{2}{3} &= (7 + \tfrac{1}{2}) \bullet \tfrac{2}{3} \\
&= 7 \bullet \tfrac{2}{3} + \tfrac{1}{2} \bullet \tfrac{2}{3} \\
&= \tfrac{14}{3} + \tfrac{1}{3} \\
&= \tfrac{15}{3} \\
&= 5
\end{aligned}
$$

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