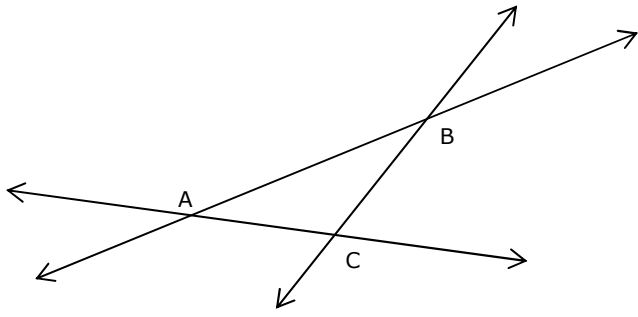# Geometry is about triangles—an overview

3 points make a triangle. A triangle's sides are the line segments connecting the points.

3 points also determine a plane, if we're talking Euclidean flatness. Keep in mind that the earth is not flat—and how messy, inconvenient and rich that makes things!

These 3 lines intersect at three points.

(The points have been labeled A, B, and C, but they could have been labeled with any capital letters.)

Polygons can be broken up into triangles
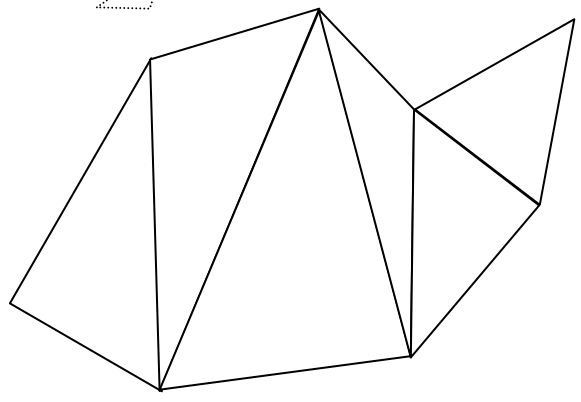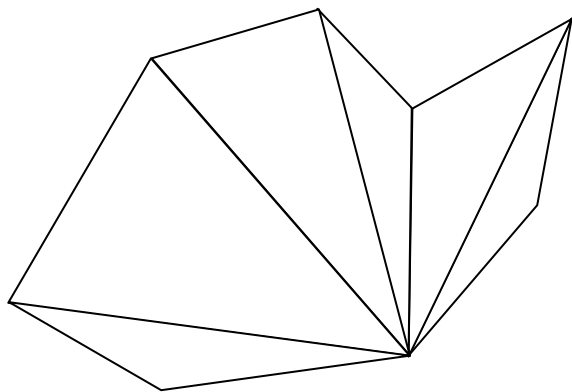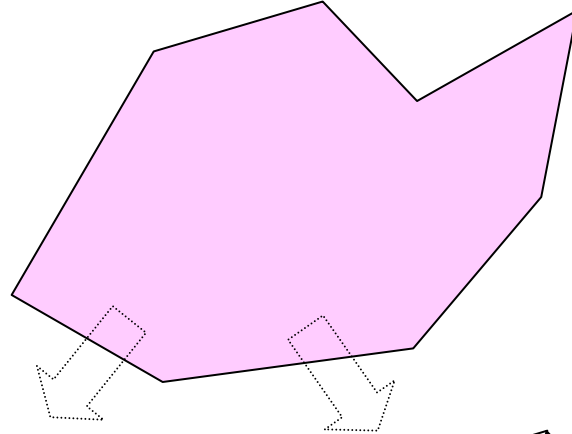
3 dimensions:

distance	length	units	inches, feet, yards, miles, centimeters, meters, kilometers
area (covering)	length x width	square units	square inches, square feet, square yards, acres, hectares
volume (filling)	length x width x height	cubic units	cups, pints, quarts, gallons, cubic centimeters, cubic inches, liters,

The **perimeter** of a figure is the **distance around it**. So the perimeter of a 3 x 5 index card

is 3 inches + 5 inches + 3 inches + 5 inches, or 16 inches.

The **area** of a figure is the number of square units it takes to **cover it**. So the area of the 3 x 5 index card is 3 inches x 5 inches, or 15 *square* inches.

$$A = bh$$

Finding the area of any **rectangle** is just like finding the area of the 3 x 5 card: you multiply the distance from left-to-right by the distance from down-to-up, or base x height.

The area of a **parallelogram** is the same as the area of a rectangle, since you can slice off a triangle from one side of the rectangle and move it to the other. It's harder to count squares, but the area has to be the same, since nothing was added or thrown away.
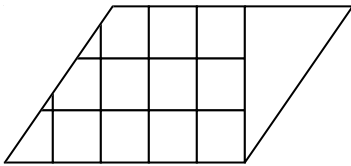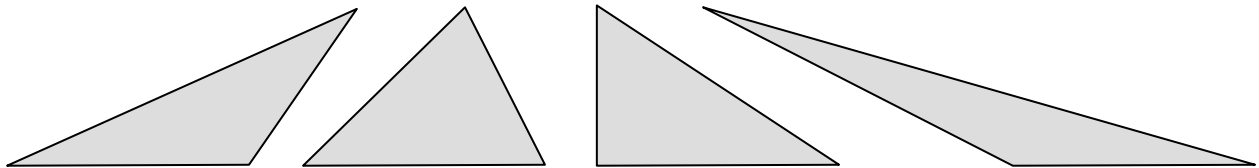
$$A = bh$$

If you slice across a parallelogram, cutting it in half, you get two identical, congruent **triangles**. (You can test this by making a cut and putting one on top of the other.)
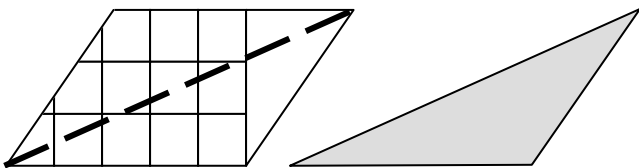
$$A = \frac{bh}{2}$$

This base x height relationship means that all the triangles below have the same area because they have the same base and the same height:
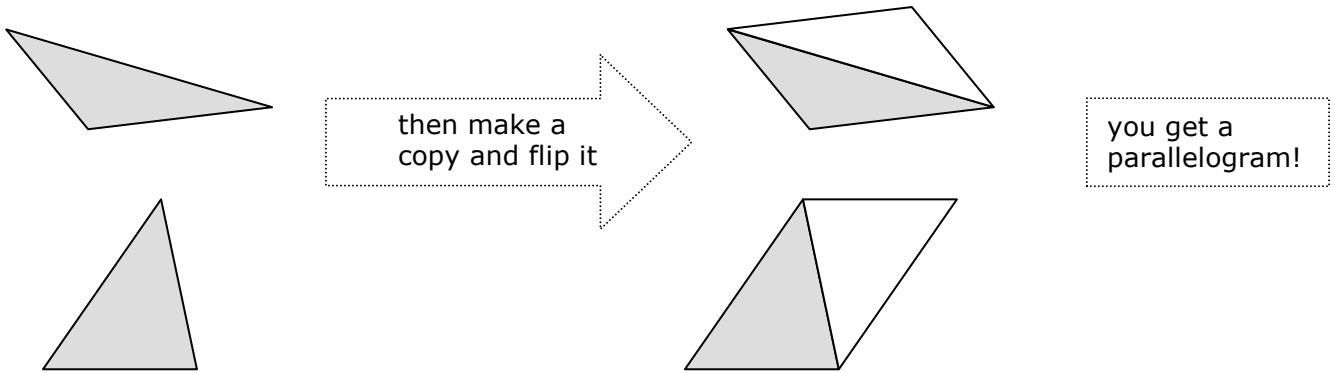
In the same way, all these parallelograms have the same area because they have the same base and the same height:
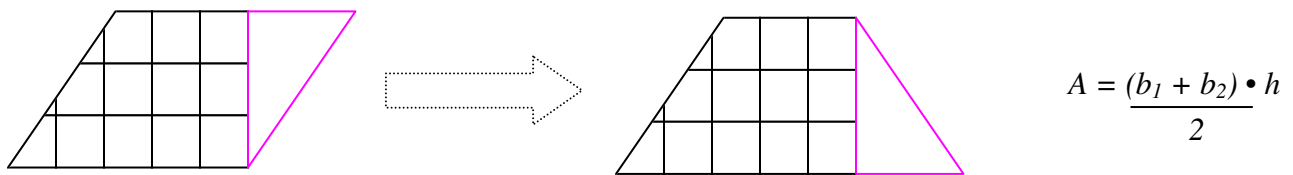
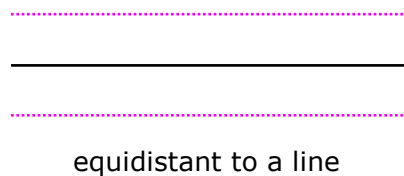For more on this, see www.soesd.k12.or.us/math/math_resources and look for index card area

Notice that you can make a parallelogram out of any triangle. Take the triangle:
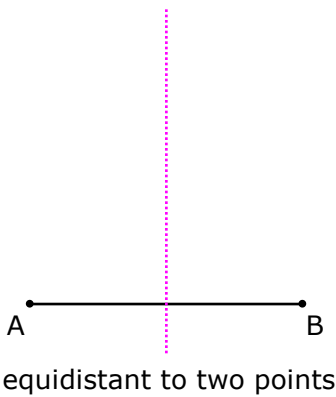
then make a
copy and flip it

you get a
parallelogram!

You can get a **trapezoid** from a parallelogram, if you just flip the triangle you sliced off the left and put on the right. The area has to be the same as the original 3 x 5 card, since (once again) nothing was added or thrown away. But we can't say $A = bh$ because the base on the bottom has gotten bigger and the base on the top has gotten smaller. But since what was subtracted from the top is what got added to the bottom, the average of the bases will be the same as the base of the original 3 x 5 card

$$A = \frac{(b_1 + b_2) \bullet h}{2}$$

collections of points

A   B

equidistant to two points

that are equidistant from two intersecting lines

P

that are the shortest distance
between a point and a line

equidistant to a line
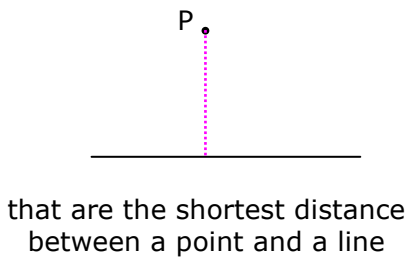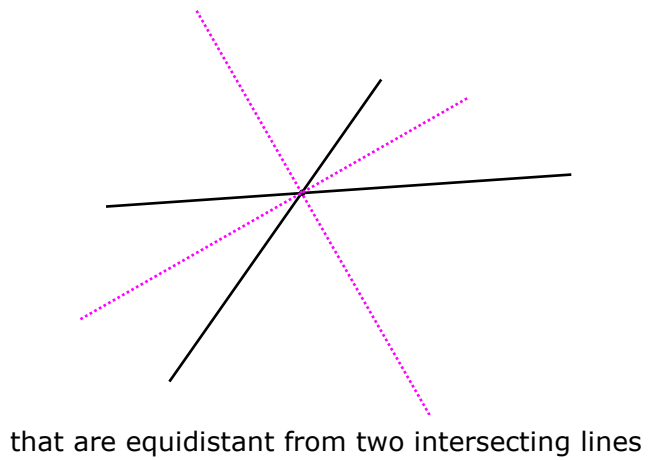
Circles can be thought of as collections of points equidistant from a single point (the center). The distance from the point to any of the points on the outside is the **radius**.

The number **pi** is the ratio of the distance around a circle to the distance across (through the center):

$$\pi = \frac{C}{d}$$

So the **circumference** of any circle (the distance around it) is equal to the diameter of the circle (the distance across it through the center).

$$C = \pi\, d$$

◄——————— *Circumference* ———————►
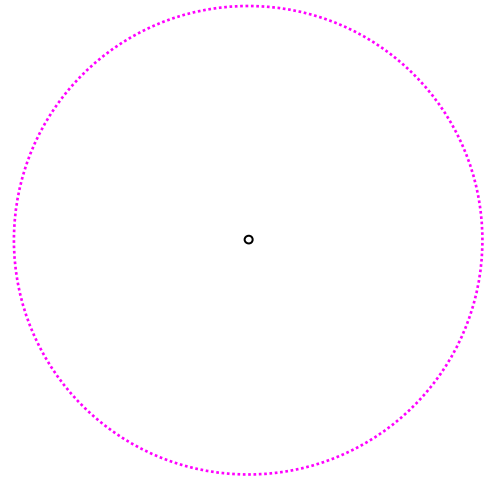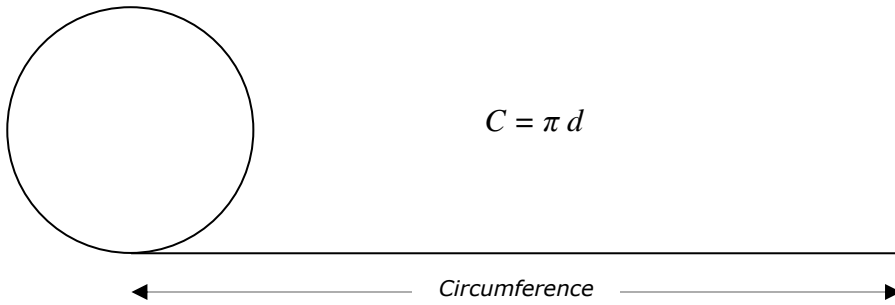
Circles can also be thought of as infinitely-many-sided polygons. Polygons are often classified by the number of their sides, a decagon, for example, is a 10-gon, a hexagon a 6-gon. So a circle could be thought of as a ∞-gon.
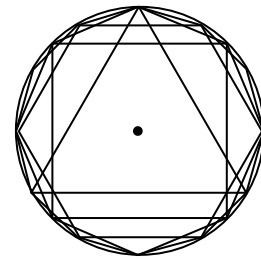
If you think of a circle as a regular polygon with a lot of (infinitely many) sides, then its area will be the sum of the areas of all the little identical triangles that make it up. As the number of sides of the polygon—and the number of its little component triangle—approaches infinity, the sum of the bases approaches the circumference of the circle and the height of each of the little triangles approaches the radius of the circle. So ultimately, if we let n = the number of sides of the circle-as-polygon, the area of any circle equals

As the number of sides increases, polygons look more and more like circles

$Area_{n\text{-}gon} = n\,(Area_{triangle})$
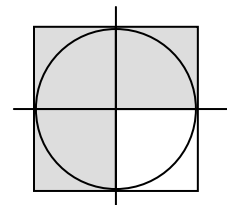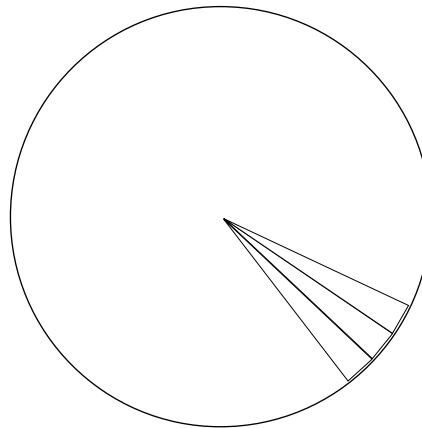
$Area_{triangle} = \dfrac{base \cdot height}{2}$

$\underset{n \to \infty}{Limit} \quad \dfrac{n \cdot bh}{2} = \dfrac{C \cdot r}{2} = \dfrac{\cancel{2}\,\pi r \cdot r}{\cancel{2}} = \pi r^2$

$\underset{n \to \infty}{Limit} \quad n \cdot b = Circumference$

$\underset{n \to \infty}{Limit} \quad height = radius$

$\underset{n \to \infty}{Limit} \quad Area_{n\text{-}gon} = Area_{circle}$

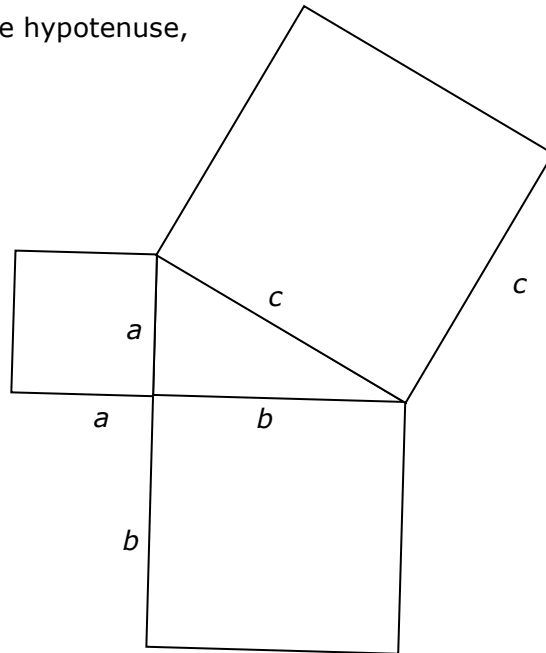$Area_{circle} = \dfrac{Circumference \cdot radius}{2} = \dfrac{2\pi r \cdot r}{2} = \pi r^2$

The area of a circle is a little more than 3 times the square of the radius
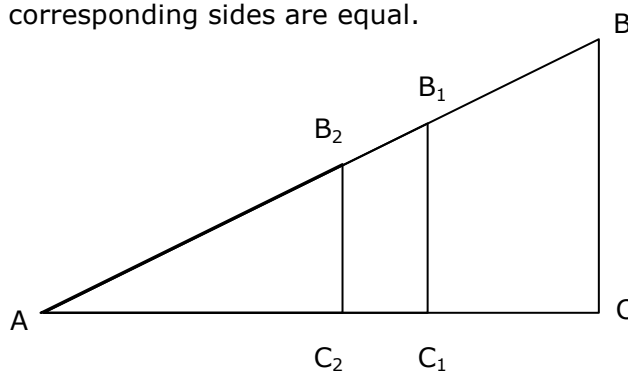
Pythagorean Theorem:
in a right triangle where a and b are legs and c is the hypotenuse,

$a^2 + b^2 = c^2$

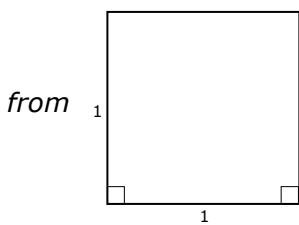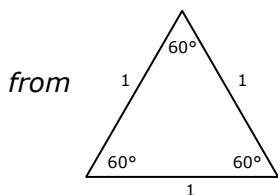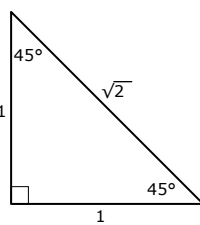In **similar triangles**, the ratios of the corresponding sides are equal.

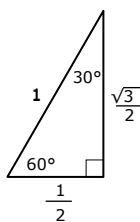$\overline{BC} : \overline{CA} = \overline{B_1C_1} : \overline{AC_1} = \overline{B_2C_2} : \overline{AC_2}$
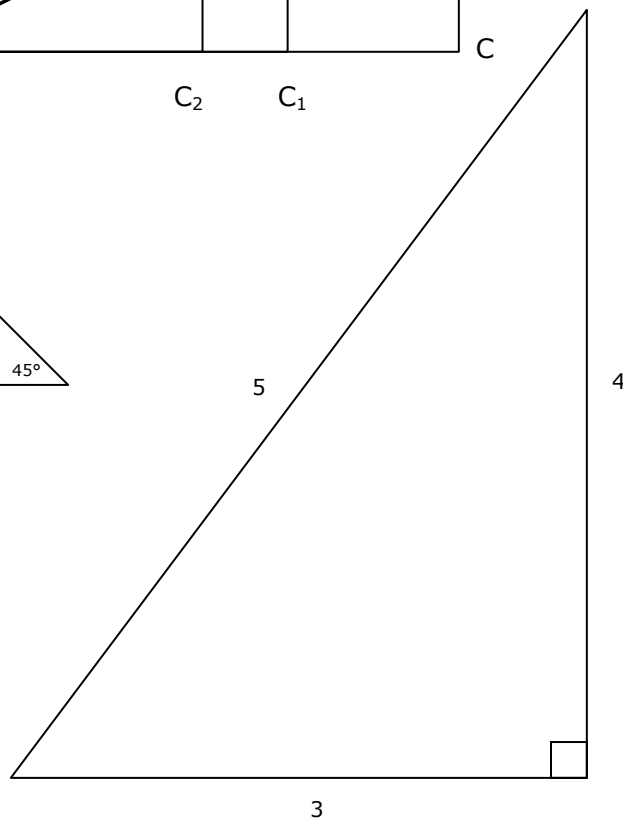
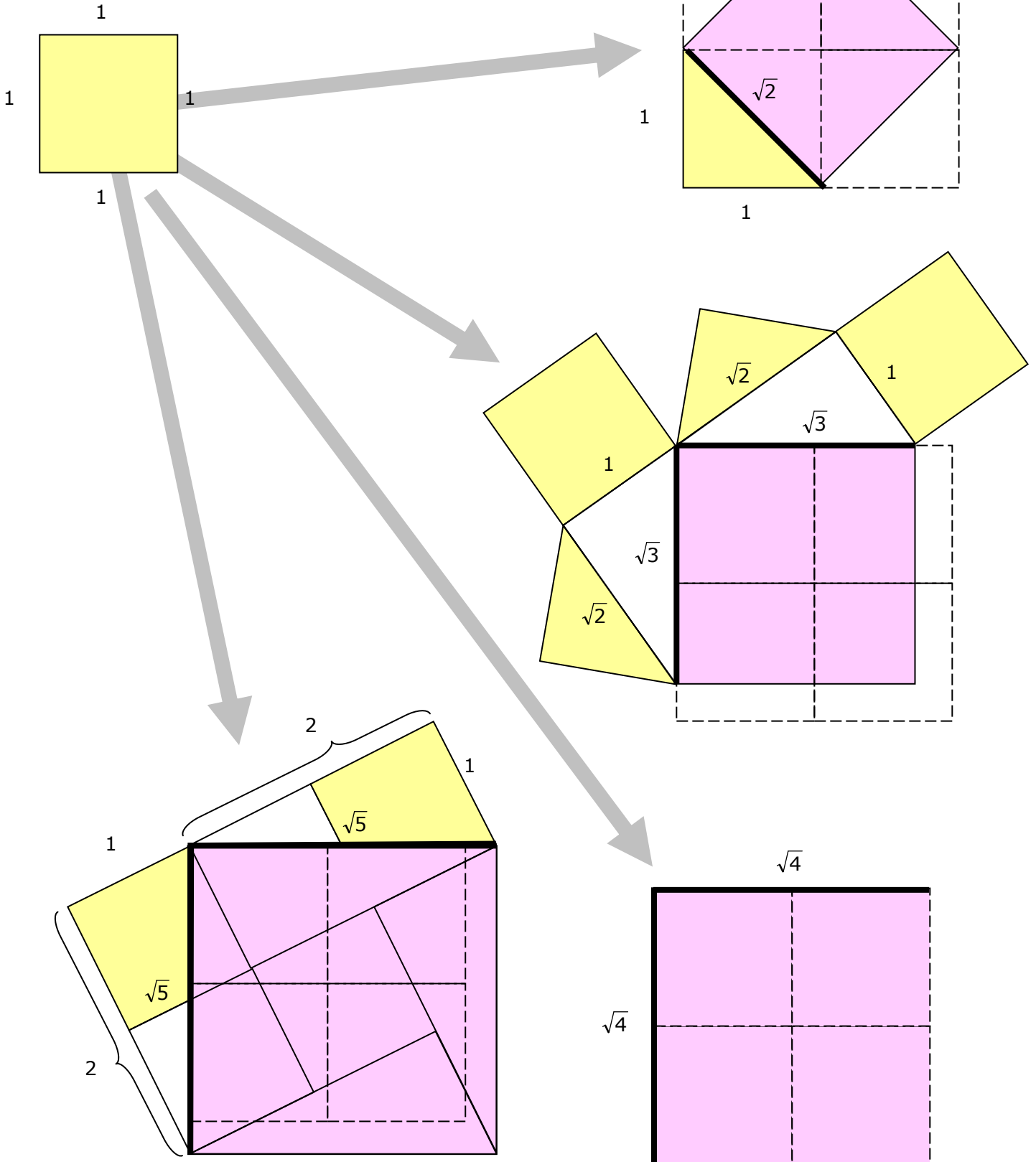**special triangles**
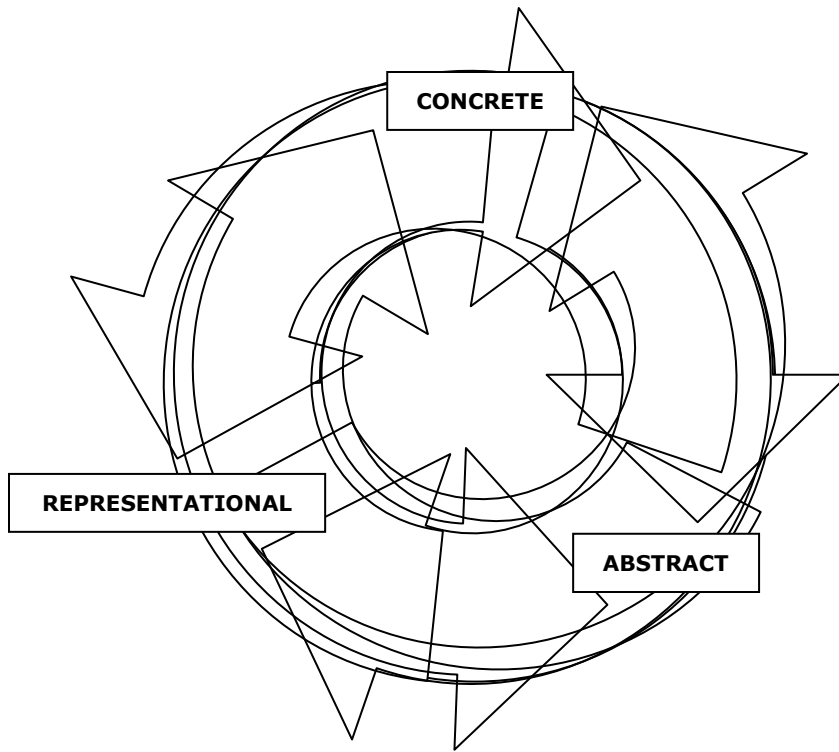
from   *you get*

from   *you get*

here's another nice one

(and also 5 x 12 x 13)

Building Square Roots and Multiples
from Unit Squares   www.soesd.k12.or.us/math/math_resources

1

1   1

1

$\sqrt{2}$

$\sqrt{2}$

1

1

$\sqrt{2}$

1

$\sqrt{3}$

$\sqrt{3}$

$\sqrt{2}$

1

2

1

$\sqrt{5}$

1

$\sqrt{5}$

2

$\sqrt{4}$

$\sqrt{4}$

CONCRETE

REPRESENTATIONAL

ABSTRACT

points

lines

triangles

squares

circles

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